Ec 453 Theories of Growth and Development I
Fall 2014, Homework On Neoclassical Growth-I

Prof. Erinc Yeldan

1. Consider the standard neoclassical growth model where output is produced along the production function: $Q = F(K, L)$, where Q is output, and K and L are the inputs of capital and labor, respectively. Labor is constant in supply. Capital depreciates at a constant rate, δ.

   (a) Derive the following expressions analytically (in algebraic form):

      i. the wage rate
      ii. the net rental rate on capital (the profit rate)
      iii. total profits per labor
      iv. share of capital and labor income in total output. Check that the sum of the shares of capital and labor income is equal to unity.

   (b) Suppose that capital labor ratio changes over time due to additions to the capital stock (per labor) net of depreciation, δ, as in the Solow model:

   $$\dot{k} = sf(k) - \delta k$$

   Using a graph of q and k, depict transitional adjustments to long run steady state and the steady state k^*. As an extension of your graph, show the factor allocation shares of output per labor among capitalists and workers, and the marginal rate of technical substitution under the optimal factor-mix.

   (c) Discus the steady state growth properties of the above neoclassical model. What makes this model, a "*neoclassical" model?*

   (d) Why is that under the standard neoclassical (Solowian) assumptions steady state per capita growth rate is zero?

2. One of the classical propositions of the Harrod-Domar model is that, for balanced growth to exist, the following relationship must hold: $v = \frac{s}{n}$; where v is the capital output ratio, s is saving rate, and n is the population growth rate. Show that the neoclassical growth model satisfies this condition by adjustments in the capital labor ratio in its adjustments towards steady state.

3. Consider two countries that are identical in their technology, population growth, and rate of depreciation. Suppose that both countries are following neoclassical adjustments towards steady state, and that they have identical saving rates. Suppose that the initial capital stocks per labor are such that country A has a higher capital-labor ratio than country B; i,e, $k_A(t_0) > k_B(t_0)$.

   (a) Argue that country B will never be richer than country A. In other words, show that the rate of per capita output growth under the transitional dynamics is monotonic. You may wish to draw a diagram if you wish.

   (b) Now suppose that saving rate in country B is increased to a higher rate than that of country A. Follow the consequences of this shift on country B's adjustment path. Discuss the per capita levels of output under the setady state. Will consumers of country B necessarily be better off?

4. Consider the standard neoclassical (Solowian) model with one good being produced by capital and labor, using a neoclassical production technology. Suppose that population growth rate is zero, and that the depreciation of capital is $\delta > 0$. Suppose that the economy is initially at steady state equilibrium.

   (a) Now suppose that due to rumors of an earthquake, the residents of this country suddenly increased their consumption and reduced their average savings rate (permanently) by half, with no other change in the remaining parameters of the economy. Do you expect any change in the steady state equilibrium of this economy? If so, discuss the effects of this reduction on per capita output, the capital labor ratio, the wage rate, profits per labor, and the profit rate under the new steady state.

   (b) Suppose now that the earthquake did indeed happen, and half of the capital stock had been destroyed. Starting from the conditions in (a) above, discuss the implications of this event for the transition dynamics and the steady state.